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HEAT TRANSFER COEFFICIENT CORRELATION FOR CONVECTIVE BOILING INSIDE PLAIN AND MICROFIN TUBES USING GENETIC ALGORITHMS.

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Abstract. *Two-phase flow heat transfer has been exhaustively studied over recent years. However, in this field several questions remain unanswered. Heat transfer coefficient prediction related to nucleate and convective boiling have been studied using different approaches, numerical, analytical and experimental. In this work, an experimental analysis, data representation and heat transfer coefficient prediction on two-phase heat transfer on nucleate and convective boiling are presented. An empirical correlation is obtained based on genetic algorithms search engine over a dimensional analysis of the two-phase flow heat transfer problem.*

keywords: *nucleate boiling, convective boiling, two-phase heat transfer, genetic algorithms*

1. Introduction

Due to a great number of industrial applications, two-phase flow heat transfer has been exhaustively studied by different research groups. Particular attention has been given to the annular flow wich includes the nucleate and convective boiling heat transfer regimes, Carey (1992), the latter being characterized by the vaporization without vapor bubles in the liquid film. Jabardo et al. (1999), Branescu (2000) and Picanç (2006) presented an extensive survey of the existing empirical and semi-empirical correlations for the two-phase flow heat transfer coefficient, h_{tp}, of these regimes. Divided into three main categories, the correlations are grouped based on the nature of the functional relation between the dimensionless numbers involved as follows: intensification models, superposition models and asymptotic models. The general form of the intensification model is based on an enhancement factor applied to the Dittus-Boelter single-phase (liquid phase) heat transfer correlation h_l. The enhacement factor (E) is a function of the Lockart-Martinelli parameter, X_{tt}.

$$h_{tp} = Eh_l \tag{1}$$

$$E = f(Xtt) \tag{2}$$

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_v}{\rho_l}\right)^{0.5} \left(\frac{\mu_l}{\mu_v}\right)^{0.1} \tag{3}$$

where x, ρ_v, ρ_l, μ_l and μ_v represent the vapor quality, the vapor mass density, the liquid mass density, the liquid dynamic viscosity and the vapor dynamic viscosity, respectively. The general superposition model is based on the assumption that the two-phase heat transfer coefficient results from the contribution of the nucleate boiling heat transfer and convective heat transfer. Chen (1966), presented a general formulation for the superposition model:

$$h_{tp} = h_{nb} + h_c \tag{4}$$

where h_{nb} and h_c represents the microscopic (nucleate boiling) and the macroscopic (bulk convective) contributions, respectively.

The asymptotic models express the two-phase heat transfer coefficient as a harmonic function of the convective boiling heat transfer coefficient and the nucleate boiling heat transfer coefficient. The general form for the asymptotic model is :

$$h_{tp}^n = h_c^n + h_{nb}^n \tag{5}$$

Using $n = 1$, Eq. (5) reduces to Eq. (4).

The known inaccuracy and dispersion of the usual correlations are mainly related to the limitations of the experimental procedure. However, most of the correlations of the models presented have their empirical coefficients derived from liner or non-linear regression procedures. Also, due to the multi-modal nature of the error function involved, minimization procedures based on gradient calculations do not guarantee that a global minimum has been reached. Local minimum attraction means inaccuracy of the coefficients.

The purpose of the present work is to carry out a dimensional analysis to develop a new correlation to predict the two-phase heat transfer coefficient. In order to build the functional relation between the dimensionless number involved, an hybrid approach based on regression analysis and stepwise regression analysis was used over an approximately 500 points databank for R-134a, R-22 and R-404, taken from publications by of Bandarra Filho (2002) and Seo and Kim (2000). The final correlations are obtained using genetic algorithm optimization package found in EES (Equation Engineering Solver) software.

2. The dimensional analysis

In order to obtain the dimensionless numbers involved in the two-phase heat transfer problem, the matrix marching procedure is used over a selected list of variables. Table 2 shows selected variables that may describe a two-phase boiling and convective heat transfer problem.

In order to take into account the two-phase flow phenomena it is plausible to consider the homogeneous hypothesis (Carey, 1992), so that the transport properties can be averanged as presented by Mac Adams et al. and Cicchcitti et al. apud Collier and Thome (1996). Here the averaging are extended to specific heat (c_p) and conductivity (k).

Equivalent mass density:

$$\rho_h = x\rho_v + (1-x)\rho_l \tag{6}$$

Equivalent viscosity (Cicchcitti et al apud Collier and Thome, 1996) :

$$\mu_h = x\mu_l + (1-x)\,\mu_v \tag{7}$$

Equivalent thermal conductivity:

$$k_h = xk_v + (1-x)\,k_l \tag{8}$$

Equivalent specific heat:

$$c_{p_h} = xc_{p_v} + (x-1)c_{p_l} \tag{9}$$

Based on the assumption that even being a two-phase flow problem, the heat transfer in nucleate and convective flow is a convective heat transfer phenomena, it is possible to analyze the problem through the Lord Raleigh dimensional approach. Thus, choosing D, U, k e μ as determinant variables and using a matrix marching procedure over the dimentional matrix, it is possible to obtain the following dimensionless numbers (case 1):

$$\begin{array}{lll} \Pi_{10} = \frac{\rho_l U_l D}{\mu_h} & \Pi_9 = \frac{\mu_h c p_h}{k_h} & \Pi_8 = \frac{i_{lv}}{U_l^2} \\ \Pi_7 = \frac{\sigma}{U_l \mu_h} & \Pi_6 = \frac{U_v}{U_l} & \Pi_5 = \frac{\rho_l U_v D}{\mu_h} \\ \Pi_4 = \frac{\delta}{D} & \Pi_3 = \frac{Dg}{U_l^2} & \Pi_2 = \frac{Dq''}{U_l^2 \mu_h} \\ & \Pi_1 = \frac{hD}{k_h} & \end{array} \tag{10}$$

Executing a base change, choosing the determinant variables as D , i_{lv}, k e μ it is possible to obtain (case 2):

$$\begin{array}{lll} \Pi_{10} = \frac{D i_{lv}^{0.5} \rho_l}{\mu_h} & \Pi_9 = \frac{\mu_h c p_h}{k_h} & \Pi_8 = \frac{U_l}{i_{lv}^{0.5}} \\ \Pi_7 = \frac{\sigma}{\mu_h i_{lv}^{0.5}} & \Pi_6 = \frac{U_l}{i_{lv}^{0.5}} & \Pi_5 = \frac{D i_{lv}^{0.5} \rho_v}{\mu_h} \\ \Pi_4 = \frac{\delta}{D} & \Pi_3 = \frac{Dg}{i_{lv}} & \Pi_2 = \frac{Dq''}{i_{lv}} \\ & \Pi_1 = \frac{Dh}{k_h} & \end{array} \tag{11}$$

Table 1: Involved variables matrix.

VARIABLE	SYMBOL	UNITY	L	M	T	Θ
Liquid mass density	ρ_l	$\frac{kg}{m^3}$	-3	1	0	0
Vapour mass density	ρ_v	$\frac{kg}{m^3}$	-3	1	0	0
Vaporisation enthalpy	i_{lv}	$\frac{J}{kg}$	2	0	-2	0
Liquidspecific heat	cp_l	$\frac{J}{kg \cdot K}$	2	0	-2	-1
Vapour specific heat	cp_v	$\frac{J}{kg \cdot K}$	2	0	-2	-1
Liquid thermal conductivity	k_l	$\frac{W}{m \cdot K}$	1	1	-3	-1
Vapour thermal conductivity	k_v	$\frac{W}{m \cdot K}$	1	1	-3	-1
Liquid dynamic viscosity	μ_l	$\frac{kg}{m \cdot s}$	-1	1	-1	0
Vapour dynamic viscosity	μ_v	$\frac{kg}{m \cdot s}$	-1	1	-1	0
Superficial tension	σ	$\frac{N}{m}$	0	1	-2	0
Vapor quality	x	-	0	0	0	0
Void fraction	α	-	0	0	0	0
Liquid film height	δ_l	m	1	0	0	0
Liquid mass velocity	G_l	$\frac{kg}{m^2 \cdot s}$	-2	1	-1	0
Vapour mass velocity	G_v	$\frac{kg}{m^2 \cdot s}$	-2	1	-1	0
Acceleration due to gravity	g	$\frac{m}{s^2}$	1	0	-2	0
Tube diameter	D	m	1	0	0	0
Microfin height	t	m	1	0	0	0
Microfin flank angle ratio	$\frac{\theta}{90^o}$	-	0	0	0	0
Microfin helix angle ratio	$\frac{\beta}{90^o}$	-	0	0	0	0
Heat flux	q''	$\frac{W}{m^2}$	0	1	-3	0
Heat transfer coefficient	h	$\frac{W}{m^2 \cdot K}$	0	1	-3	-1

And executing another base change, and using D, U_l, k e q", follows that(case 3):

$$\begin{array}{ccc} \Pi_{10} = \frac{U_l^3 \rho_l}{q''} & \Pi_9 = \frac{D q'' cp_h}{U_l^2 k_h} & \Pi_8 = \frac{i_{lv}}{U_l^2} \\ \Pi_7 = \frac{U_l \sigma}{D q''} & \Pi_6 = \frac{U_v}{U_l} & \Pi_5 = \frac{U_l^3 \rho_v}{q''} \\ \Pi_4 = \frac{\delta}{D} & \Pi_3 = \frac{D g}{U_l^2} & \Pi_2 = \frac{D q''}{U_l^2 \mu_h} \\ & \Pi_1 = \frac{h D}{k_h} & \end{array} \tag{12}$$

These three sets of dimensionless numbers can represent the problem of heat transfer during nucleate and convective boiling. However the dimensional analysis does not give the functional relation between them. It is necessary to use different tools to build this functional relation. It is possible to identify some usual dimensionless numbers like:

$$\Pi_1^{(1,2,3)} = \frac{hD}{k_h} \textit{ (nusselt number)} \tag{13}$$

$$\Pi_{10} = \frac{DU_{l,v}\rho_h}{\mu_h} \text{ (liquid or vapor reynolds number)} \tag{14}$$

$$\Pi_9 = \frac{\mu_h cp_h}{k_h} \text{ (homogeneous prandtl number)} \tag{15}$$

3. The functional relation

Using a hybrid procedure of regression analysis and stepwise regression analysis it is possible to derive a functional relation based on the correlation coefficients of the dimensionless numbers involved. Stephan and Abdelsalam (1980) used similar procedure to derive a functional relation for the free convection boiling problem. In order to investigate the regular dimensionless numbers presented in the literature, the "case 1" is the object of this paper.

At first, an ordinary linear regression is obtained for each individual dimensionless number, using the functional relation:

$$\Pi_1 = C_1 + C_2\Pi_n^{C_3} \tag{16}$$

Resulting in the following correlation coefficients:

Table 2: Residuals and correlation factor.

	Π_{10}	Π_9	Π_8	Π_7	Π_6	Π_5	Π_4	Π_3	Π_2
Res	46.49	39.35	58.6	48.45	31.42	22.35	-	48.26	45.75
R^2	0.25	57.85	4.75	4.89	20.65	75.08	-	1.65	4.39

From the Tab. (2), the best fit dimensionless number was Π_5, followed by the number Π_9 and Π_6, wich gave the lowest residuals and highest correlation factor. The dimensionless numbers $\Pi_5 = \frac{\rho_l U_v D}{\mu_h}$, $\Pi_9 = \frac{\mu_h cp_h}{k_h}$ and $\Pi_6 = \frac{U_v}{U_l}$ represent the modified reynolds number for the vapour phase, the modified prandtl number and the slip factor, respectively. Unfortunately, there are no void fraction or film thickness measurements in the experimental data used in this work and thus, the dimensionless numbers where the film thicknesses are involved will be neglected. The following two different kinds of functional relations between the dimensionless numbers are considered:

$$\Pi_1 = C_1 + C_2\Pi_5^{C_3}\Pi_i^{C_4} \textit{ with } i \neq 1,5 \textit{ type 1} \tag{17}$$

$$\Pi_1 = C_1 + C_2\Pi_5^{C_3} + C_4\Pi_i^{C_5} \textit{ with } i \neq 1,5 \textit{ type 2} \tag{18}$$

Results:

Table 3: Residuals and Bias.

	Π_9		Π_{10}		Π_8	
	Eq. (17)	Eq. (18)	Eq. (17)	Eq. (18)	Eq. (17)	Eq. (18)
Res	20.66	25.91	21.79	21.81	21.98	22.00
Bias	89.27	-14.66	96.41	97.30	89.7	94.78
	Π_7		Π_6		Π_5	
	Eq. (17)	Eq. (18)	Eq. (17)	Eq. (18)	Eq. (17)	Eq. (18)
Res	21.77	58.60	21.34	22.35	21.86	29.85
Bias	41.37	287.45	34.82	36.34	41.51	118.32
	Π_3		Π_2		Π_4	
	Eq. (17)	Eq. (18)	Eq. (17)	Eq. (18)	Eq. (17)	Eq. (18)
Res	22.98	22.85	22.09	22.33	-	-
Bias	43.35	58.34	43.95	45.57	-	-

where:

$$Res = \frac{1}{N}\sum_{i=1}^{N}\left|\Pi_1^{exp} - \Pi_1^{calc}\right| \tag{19}$$

$$Bias = \frac{1}{N}\sum_{i=1}^{N}\left(\Pi_1^{exp} - \Pi_1^{calc}\right) \tag{20}$$

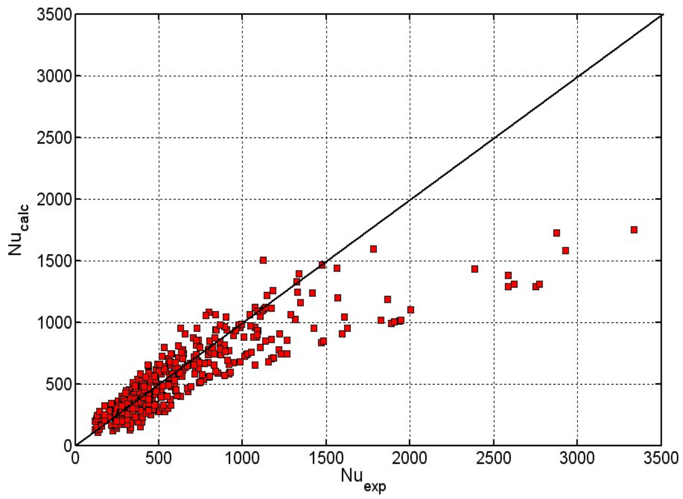

Figure 1: Experimental nusselt number versus Calculated nusselt number functional relation type 1

Based only on the results presented in Tab. (3) it is not possible to find the best fit functional relation. It is also important to analyze the derived coefficients and exponents,in order to select dimensionless numbers which shares data representation. Also, smaller coefficient dimensionless numbers are neglected.

Figure (1) shows the results for a linear regression of the Π_5 and Π_9 numbers using a functional relation of type "Eq. 17".

Proceeding with this marching the following functional relation is derived:

$$\Pi_1 = C_1 + C_2 \Pi_5^{C_3} \Pi_9^{C_4} \Pi_6^{C_5} \Pi_2^{C_6} \tag{21}$$

3.1. Geometric Factors

The geometric factor for the microfin tube problem are include in the functional relation as an enhancement factor in the form $(1 + F)$. Where:

$$F = \alpha_1 \beta_r^{\alpha_2} \theta_r^{\alpha_3} \left(\frac{t}{D}\right)^{\alpha_4} \tag{22}$$

And the final relational function is presented as follows:

$$\Pi_1 = \left[C_1 + C_2 \Pi_5^{C_3} \Pi_9^{C_4} \Pi_6^{C_5} \Pi_2^{C_6}\right](1 + F) \tag{23}$$

4. Correlation based on genetic algorithm minimization

The basic functional relation being derived, it is now necessary to find the coefficients that best fit the databank. Usually, it is common to use a linear regression analysis or any other gradient driven minimization method as a search method to find the correlation coefficients. However, as pointed out by Hacker at all (2002), this procedure, in multi-modal minimization problems, is attracted to local minimums and it is initial trial dependent. Therefore, it does not guarantee that a global minimum is reached. In order to avoid this problem, in this work, a genetic algorithm minimization procedure is used. Based on a public domain genetic optimization method called PIKAIA (Charbonneau,2002), the EES software has a genetic algorithm optimization function implemented. This EES feature was used to minimize the total residual function:

$$Res_t = \sum_{i=1}^{N} \left|\Pi_1^{exp} - \Pi_1^{calc}\right| \tag{24}$$

The final functional relation is then correlated to the databank, firstly only the plain tube data and then including the microfin tube data. The search space was delimited based on the previous experience of the functional relation construction, the closest highest interger value being used as delimeters. For example, the exponents for the dimensionless numbers in the previous stage were between -2 and 2, and thus, in order to expand the search space, the boundaries where defined as ± 5. For the genetic algorithm search engine setup, by default, the number of individuals and generations were set to 64 and 512 respectively and the mutation rate was set to aproximatly 0.47. After 5 working days, a direct search method using the genetic algorithm output as the initial tryout was proceeded for refinement, and the stop criteria was set to 10^{-4}. The coefficients for the first correlation Eq. (21) are:

$$\begin{array}{lll} C_1 = 37.14 & C_2 = 0.1226 & C_3 = 0.6818 \\ C_4 = 0.3381 & C_5 = 0.2771 & C_6 = 0.1685 \end{array} \tag{25}$$

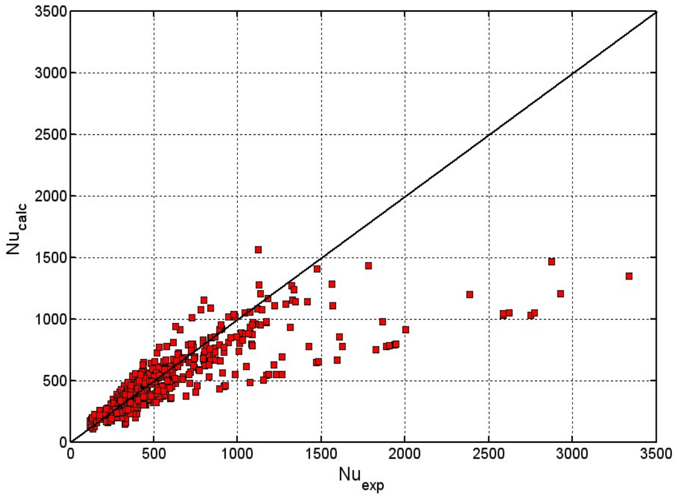

Figure 2: Experimental nusselt number versus calculated nusselt number using genetic algorithm minimization for plain tube data.

Figure (2) shows the relation between the calculated dimensionless Nusselt number Π_1 and the experimental data for the plain tube. The residual is 21.14 and bias is 24.6. Including the microfined tube experimental data and using the functional relation in Eq. (23) , the coefficients are:

$$\begin{array}{lll} C_1 = 37.14 & C_2 = 0.1226 & C_3 = 0.6818 \\ C_4 = 0.3381 & C_5 = 0.2771 & C_6 = 0.1685 \\ \alpha_1 = 1.364 & \alpha_2 = 0.4111 & \alpha_3 = 0.9627 \\ & \alpha_4 = 1.603 & \end{array} \tag{26}$$

For these derived correlations the valid usage range is:

- Fluids: R-22, R-134a and R-404
- G: 150 up to $250 kg/m^2 s$
- q: 4.7 up to $14.2 W/m^2$
- x: 0.15 up to 0.75

Figure (3) shows the relation between the calculated dimensionless number Π_1 and the experimental data for the whole databank, with approximately 500 experimental points (where 400 points for plain tubes)(Bandarra Filho, 2002) (Seo and Kim, 2000). Bandarra Filho (2002) reported a residual for his plain tube experimental data of 17% for a mass velocity below 200 $\frac{kg}{m^2 \cdot s}$ and 60.04% above this limit. Figure (4) summarizes the results from four similar correlations and the correlation proposed here. Two correlations developed by Bandarra Filho (2002), one by Dengler and Addoms (1956) and an other by Penek apud Bandarra Filho (2002). It is possible to observe that the correlation here presented results in less residual data dispersion.

Bardarra Filho (2002):

$$\frac{h_b}{h_l} = 1 + 3.0 \cdot X_{tt}^{-0.65} \cdot Bo^{0.23} \tag{27}$$

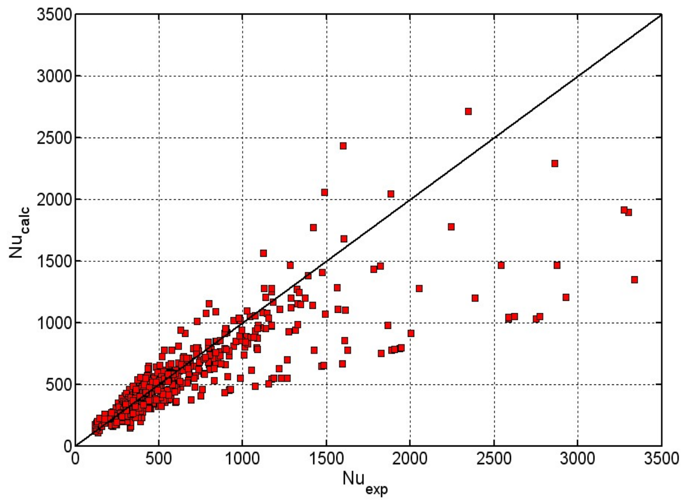

Figure 3: Experimental nusselt number versus calculated nusselt number using genetic algorithm minimization with geometric factors

$$\frac{h_b}{h_l} = 1 + 0.74 \cdot Bj^{2/3} \cdot Fr_l^{1/3} \tag{28}$$

where, $Bo = \frac{q"}{Gi_{lv}}$, $Bj = \frac{q''D}{k_l T_{sat}}$ and $Fr_l = \frac{G(1-x)}{\rho_l^2 gD}$.

The valid usage range is:

- Fluids: R-12, R-22, R-134a and R-404
- G: 25 up to $700 kg/m^2s$
- x: 0.01 up to 0.99

Dengler e Addoms (1956):

$$\frac{h_b}{h_l} = 3.5 \cdot X_{tt}^{-0.5} \tag{29}$$

The valid usage range is:

- Fluid: water
- G: 55 up to $1100 kg/m^2s$
- x: 0 up to 0.7

Panek (1992) apud Bandarra Filho (2002):

$$\frac{h_b}{h_l} = 3.686 \cdot X_{tt}^{-0.563} \tag{30}$$

The valid usage range is:

- Fluids: R-12 and R-134a
- G: 100 up to $500 kg/m^2s$
- x: 0.20 up to 0.60

Table 4: Residuals

	Work	Dengler e Addons	Bandarra Filho (1)	Bandarra Filho (2)	Paneck
Res [%]	19.62	79.83	30.23	65.74	28.56

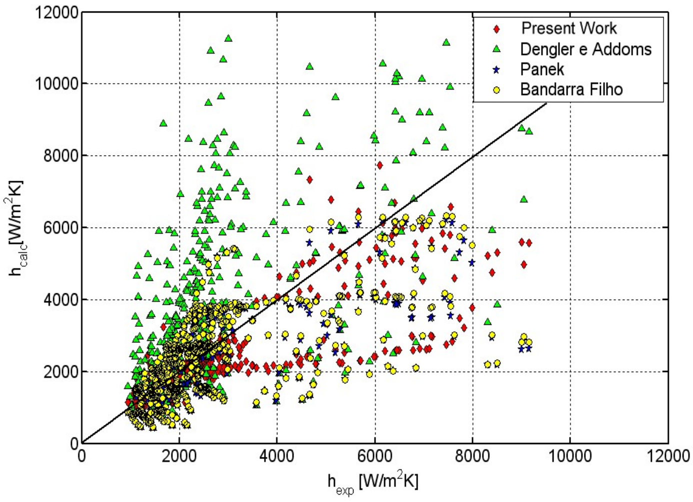

Figure 4: Experimental versus calculated heat transfer coefficient for different correlations

5. Conclusion

This present work demonstrated a procedure to derive the functional relation between dimensionless numbers in convective and nucleate boiling heat transfer. It also proposes the use of genetic algorithms to correlate the derived functional relation to the experimental data. The results obtained are satisfactory in comparison to the results previously reported. Further research is necessary to identify the new dimensionless numbers revealed in cases 2 and 3 and also investigate their physical meaning and correlation to experimental data.

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